Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, December - 2024 /January - 2025
MATHEMATICS - IV
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AE)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A **(25 Marks)**

1.a) Write C-R equations in Cartesian as well as in polar form. [2]
b) Define an analytic function. Explain the Milne-Thomson method. [3]
c) Evaluate $\oint_C \frac{3z^2+7z+1}{z+1}dz$ where C is $|z| = \frac{1}{2}$. [2]
d) Define a simple pole and a pole of order m. [3]
e) If $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \frac{\pi}{2}$ then find the value of $\int_{-\pi}^{\pi} \frac{d\theta}{5-3\cos\theta} = ?$ [2]
f) Find the invariant points of the transformation $w = \frac{1+iz}{1-iz}$. [3]
g) Define the Fourier series expansion of *f(x)* in the interval (*0, 2l*). [2]
h) If the Fourier sin transform of a function *f(x)* is $\frac{1-\cos n\pi}{n^2\pi^2}, 0 \le x \le \pi$, find *f(x)*. [3]
i) Classify the differential equation $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$. [2]
j) Solve $2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$ by the method of separation of variables. [3]

PART – B

(50 Marks)

2.a) Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.
b) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f^1(z)|^2$. [5+5]

OR

3.a) Show that $w = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$ is an analytic function and determine $\frac{dw}{dz}$.
b) In a two dimensional fluid flow, the stream function ψ is given by $\psi = \frac{-y}{x^2+y^2}$ find the velocity potential ϕ. [5+5]

4.a) Evaluate $\oint_C (z-a)^{-1}\,dz$ where C is a simple closed curve containing the point z=a (i) outside C, and (ii) inside C.

b) Find the first four terms of the Taylor's series expansion of the complex viable function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about z=2. Find the region of convergence. [5+5]

OR

5.a) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^z}{\left(z^2+\pi^2\right)^2}$ where C is the circle $|z| = 4$.

b) Find the Laurent's series expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ about z=0 in the region $2 < |z| < 3$. [5+5]

6. Using Cauchy's residue theorem, evaluate $\int_{-\pi}^{\pi} \frac{d\theta}{5+4\sin\theta}$. [10]

OR

7. Find the bilinear transformation which maps the points $(-1,0,1)$ into the points $(0,i,3i)$. Also find its invariant points. [10]

8. Obtain a half range cosine series for $f(x) = \begin{cases} kx & ; 0 \le x \le l/2 \\ k(l-x); & l/2 \le x \le l. \end{cases}$.

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..$ [10]

OR

9. Find the Fourier transform of $f(x) = \begin{cases} 1, for\ |x| < 1 \\ 0, for\ |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$. [10]

10. Solve the one-dimensional wave equation by the method of separation of variables and hence find all the possible solutions. [10]

OR

11. A bar AB of length 10 cm has its ends A and B kept at 30^o and 100^o temperatures respectively, until steady-state condition is reached. Then the temperature at A is lowered to 20^o and that at B to 40^o and these temperatures are maintained. Find the subsequent temperature distribution in the bar. [10]

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